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MELTING OF A SOLID WITH ACCOUNT FOR THE CHANGE IN DENSITY

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The two-dimensional problem of the melting of a solid with account for the jump in the density in the phase transition is solved numerically in the stationary formulation.

We consider the problem of the melting of a solid with a rectangular cross section. Melting occurs under the effect of the heat fluxes from the boundary. Problems involving phase transitions are considered in various formulations in a number of works [1-3]. It is usually assumed that the density is continuous in the phase transition, otherwise the problems of describing the processes of heat and mass transfer become substantially more involved [4]. In the present work we will account for the change in density in the transition from the solid state to the liquid one with consideration of the stationary thermal state within the Stefan approximation. The situation with a higher density for the melt, which is characteristic for a wide range of materials, is considered. This situation is most typical for the melting of porous media.

We consider the stationary thermal state of the solid being melted, with the initial cross section given by the expression

$$\Omega = \left\{ x \mid x = (x_1, x_2),\ 0 < x_\alpha < l_\alpha,\ \alpha = 1, 2 \right\}.$$

The left boundary will be considered to be maintained at a constant temperature that is higher than the temperature of the phase transition. The temperature at the right boundary is also constant but in this case it is lower than the temperature of the phase transition. The upper and lower boundaries are considered to be heat-insulated.

The solid melts in the vicinity of a hot wall, and with account for the higher density of the liquid phase, the melt takes only a portion of the freed region. The density of the melt is higher than that of the porous medium, and therefore the melt gathers in the lower portion of the region in the course of melting (the effect of gravity). Therefore, the processes of heat transfer should be considered in three phases: solid, liquid, and gaseous (the air).

The stationary thermal state is described by the heat conduction equation as follows:

$$\sum_{\alpha=1}^{2} \frac{\partial}{\partial x_\alpha} \left(k(x, u) \frac{\partial u}{\partial x_\alpha} \right) = 0, \tag{1}$$

where u is the temperature, k is the heat conduction coefficient. We restrict ourselves to the case where the heat conduction coefficient is constant within the solid, liquid, and gaseous phases. The lower and upper boundaries are heat-insulated, and therefore

$$\frac{\partial u}{\partial x_2} = 0, \quad x_2 = 0, \quad l_2, \quad 0 < x_1 < l_1. \tag{2}$$

At the right and left boundaries we have

$$u = u_l, \quad x_1 = 0, \quad 0 < x_2 < l_2, \tag{3}$$

$$u = u_r, \quad x_1 = l_1, \quad 0 < x_2 < l_2, \tag{4}$$

with $u_l > u^*$, $u_r < u^*$, where $u*$ is the phase transition temperature.

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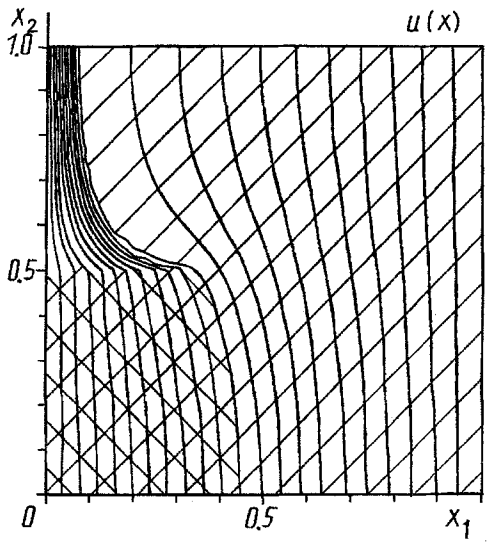
TABLE 1. Dependence of the Melt Height on the Ratio of Densities

H	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
κ	0.29	0.49	0.63	0.72	0.76	0.82	0.85	0.88	0.92



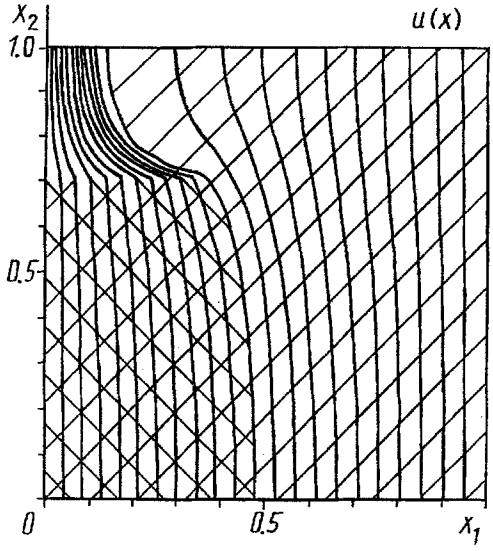


Fig. 1

Fig. 2

Within the Stefan approximation employed the phase transition boundary S is defined as follows:

$$S = \left\{ x \mid x \in \Omega, \quad u(x, t) = u^* \right\}. \tag{5}$$

At the boundaries of the phases the usual conditions of ideal contact are assumed: the temperature and the heat fluxes are continuous.

The separation of the region outside the solid is modeled as follows. Let the initial region of computations Ω be divided into two subregions Ω^+ (where $u > u^*$) and Ω^- (where $u < u^*$, which corresponds to the region taken by the melt, and the air). Let H be the level of the melt, and then the two subregions within Ω^+ can be separated:

$$\Omega^+ = \Omega_m \cup \Omega_a,$$

$$\Omega_m = \left\{ x \mid x \in \Omega^+, \quad x_2 \le H \right\},$$

$$\Omega_a = \left\{ x \mid x \in \Omega^+, \quad x_2 \ge H \right\},$$

i.e., Ω_m is the region taken by the melt, and Ω_a is correspondingly taken by the air.

We denote the ratio of the densities of the solid and the melt by κ. Then, neglecting the density of the air, we obtain

$$\kappa = \text{meas}\,(\Omega_m)/\text{meas}\,(\Omega^+), \tag{6}$$

where meas (Ω) is the area of the region Ω. This relationship can be considered to be a condition for the determination of the melt height H. Within each of the media the thermophysical parameters are different, and therefore

$$k(x, u) = \begin{cases} k_p, & x \in \Omega^-, \\ k_m, & x \in \Omega_m, \\ k_a, & x \in \Omega_a. \end{cases} \tag{7}$$

Equation (1) with conditions (2)-(6) describes completely the temperature fields in melting of the solid, which initially occupied the region Ω.

We formulate the problem in dimensionless variables. For the dimensionless variables the same notation is used as for the dimensional one. We put the variables in dimensionless form on l_1 and the parameters of the solid state, and we define the dimensionless temperature by $(u - u^*)/(u_l - u_r)$. In the dimensionless variables equation (1) and boundary and initial conditions (2), (5) remain the same, whereas (3) and (4) give

$$u = \eta\,, \quad x_1 = 0\,, \quad 0 < x_2 < l_2\,, \tag{8}$$

$$u = \eta - 1\,, \quad x_1 = 1\,, \quad 0 < x_2 < l_2\,. \tag{9}$$

The problem posed is characterized by the parameter l_2 and the corresponding (relative) thermophysical parameters for the melt and the air.

Our main concern will be with problems with a predetermined level of the melt H. In this case the melt density is calculated in accordance with condition (6). Difference methods [5] are used for the numerical solution of problem (2), (5), (7)-(9). To refine the phase boundaries the simplest iteration process of successive refinement [6] of the phase transition boundary is used. Elliptic grid problems are solved by an iterative alternate-triangle method of approximate factorization, i.e., by the method of conjugate gradients [7, 8].

As the main version, the problem was considered with $k_m = 1$, $k_a = 0.05$, $l_2 = 1$, and $\eta = 0.5$. The dependence of the melt level H on κ for the given version is presented in Table 1. Isotherms for the case when $H = 0.5$ are presented in Fig. 1, and the analogous data for the problem with $H = 0.7$ are shown in Fig. 2. The calculations were performed on a uniform rectangular grid with the dimensions 101×101. The roles of the thermophysical parameters (heat capacity coefficients k_m and k_a) and the geometry (the parameter l_2) were studied. The problem was considered also under conditions of more general boundary thermal regimes than those given by (2)-(4).

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